6. Mass Transfer

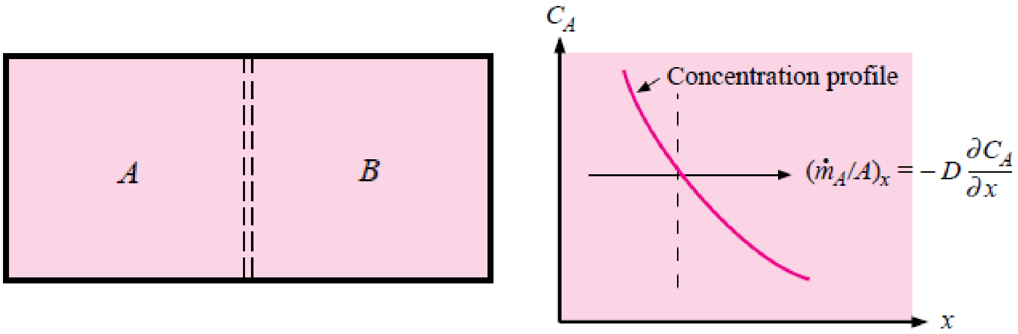


Figure 1 Diffusion of component *A* into component *B*.

Fick's Law of Diffusion:

$$\frac{\dot{m}_A}{A} = -D\frac{\partial C_A}{\partial x}$$

where,
D = proportionality constant–diffusion coefficient, m^2/s
$\dot{m}_A$ = mass flux per unit time, kg/s
C_A = mass concentration of component *A* per unit volume, kg/m^3

Analogy with Heat and Momentum Diffusion:

Fourier law of heat conduction: $\left(\frac{\dot{Q}}{A}\right)_x = -k\frac{\partial T}{\partial x}$

Newton's viscosity law: $\tau = \mu\frac{\partial u}{\partial y}$

The Mass Transfer Coefficient:

$$\dot{m}_A = K\,A(C_{A_1} - C_{A_2}) = -\frac{D\,A(C_{A_2} - C_{A_1})}{\Delta x}$$

$$K = \frac{D}{\Delta x}$$

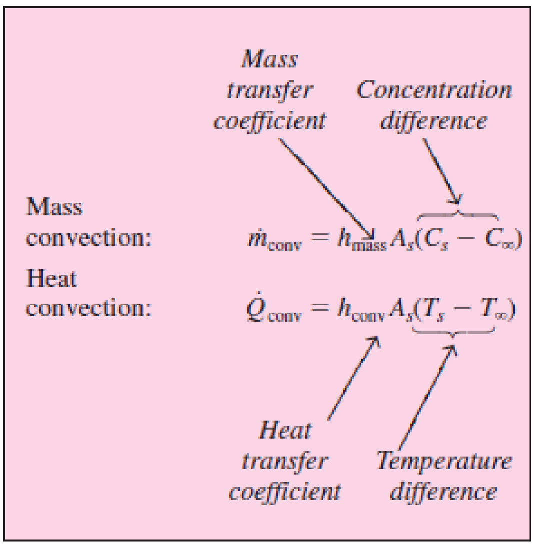
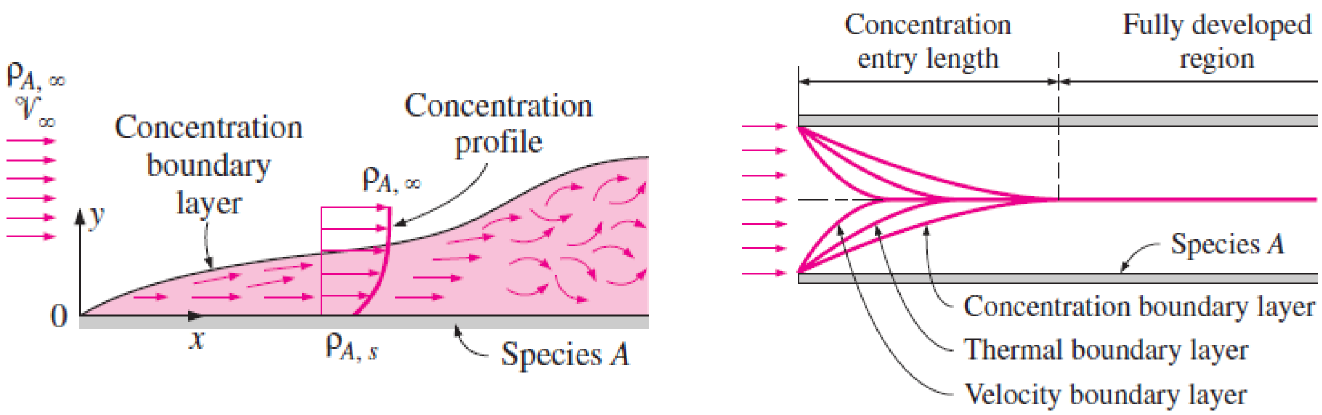


Figure 2 Analogy between convection heat transfer and convection mass transfer.

Mass Convection:



(a) external flow (b) internal flow

Figure 3 The development of a concentration boundary layer.

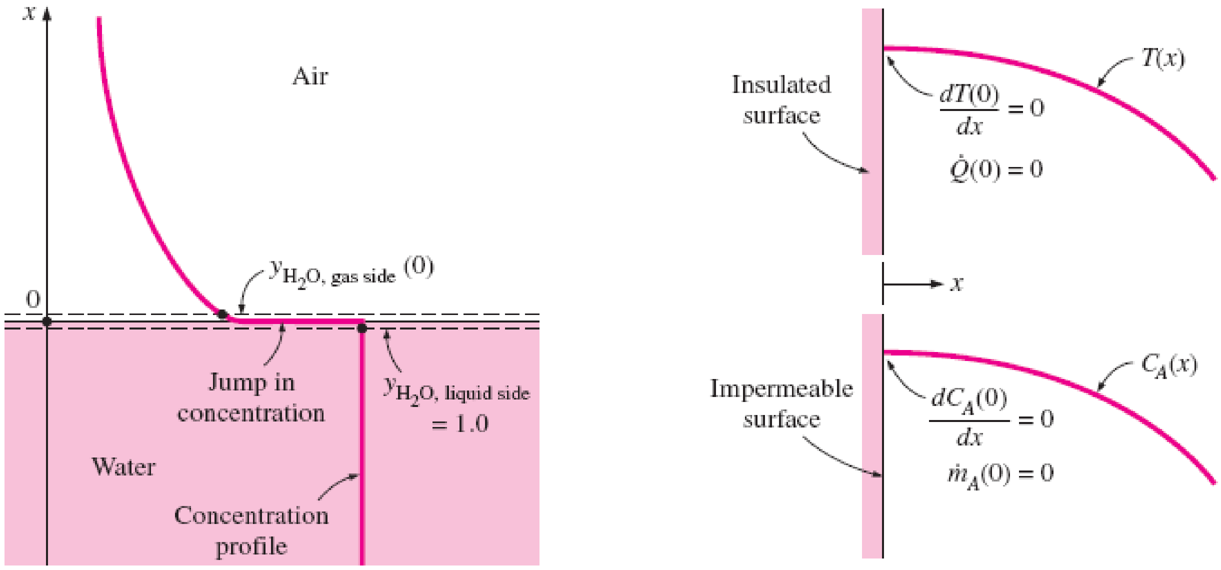
The Governing Equations:

Momentum Equation: $u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=\nu\dfrac{\partial^2 u}{\partial y^2}$

Energy Equation: $u\dfrac{\partial T}{\partial x}+v\dfrac{\partial T}{\partial y}=\alpha\dfrac{\partial^2 T}{\partial y^2}$

Concentration Equation: $u\dfrac{\partial C_A}{\partial x}+v\dfrac{\partial C_A}{\partial y}=D\dfrac{\partial^2 C_A}{\partial y^2}$

Boundary Conditions:



(a) Unlike temperature, the concentrations of species on the two sides of a liquid–gas (or solid–gas or solid–liquid) interface are usually not the same.

(b) An impermeable surface in mass transfer is analogous to an insulated surface in heat transfer.

Figure 4 Two common types of boundary conditions.

The Schmidt Number:

$$Sc = \frac{\nu}{D} = \frac{\mu}{\rho D}$$

Note:
The Schmidt number plays a role similar to that of the Prandtl number in convection heat transfer problems ($\Pr = \frac{\upsilon}{\alpha}$).

The Lewis Number:

$$Le = \frac{\alpha}{D} = \frac{Sc}{\Pr}$$

The velocity, thermal and concentration laminar boundary thickness:

$$\frac{\delta_V}{\delta_T} = \Pr^n \qquad \frac{\delta_V}{\delta_C} = Sc^n \qquad \frac{\delta_T}{\delta_C} = Le^n$$

Note:
$n = \frac{1}{3}$ for most applications in all three relations.

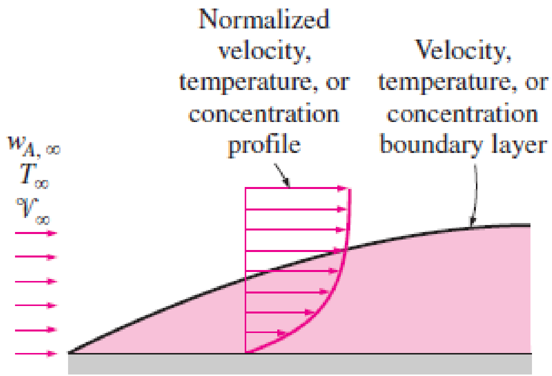


Figure 5 when the molecular diffusivities of momentum, heat, and mass are equal to each other, the velocity, temperature, and concentration boundary layers coincide.

The Sherwood Number:

$$Sh = \frac{K(h_{mass})x}{D}$$

Note:
The Sherwood number plays a role similar to that of the Nusselt number in convection heat transfer problems:

$$Nu = \frac{h\,x}{k} = f(\mathrm{Re}_x, \Pr)$$

$$Sh = \frac{K(h_{mass})\,x}{k} = f(\mathrm{Re}_x, Sc)$$

Chilton-Colburn Analogy for Mass Transfer:

Heat transfer Stanton Number: $$St = \frac{Nu}{Pe} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} = \frac{h}{\rho C_p U_\infty}$$

<u>Mass transfer Stanton Number:</u> $$St_{mass} = \frac{Sh}{\mathrm{Re}\,Sc} = \frac{K(h_{mass})}{U_\infty}$$

$$St\,\mathrm{Pr}^{2/3} = \frac{C_f}{2} = St_{mass}\,Sc^{2/3}$$

Consequently,

$$\frac{St}{St_{mass}} = \left(\frac{Sc}{\mathrm{Pr}}\right)^{2/3}$$

$$\frac{h}{K(h_{mass})} = \rho C_p \left(\frac{Sc}{\mathrm{Pr}}\right)^{2/3} = \rho C_p \left(\frac{\alpha}{D}\right)^{2/3} = \rho C_p\, Le^{2/3}$$

Mass Convection Correlations:

Table 1 Sherwood number relations in mass convection for specified concentration at the surface corresponding to the Nusselt number relations in heat convection for specified surface temperature

Convective Heat Transfer		Convective Mass Transfer	
1. *Forced Convection over a Flat Plate*			
(*a*) Laminar flow ($\mathrm{Re} < 5 \times 10^5$)			
$\mathrm{Nu} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$,	$\mathrm{Pr} > 0.6$	$\mathrm{Sh} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Sc}^{1/3}$,	$\mathrm{Sc} > 0.5$
(*b*) Turbulent flow ($5 \times 10^5 < \mathrm{Re} < 10^7$)			
$\mathrm{Nu} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$,	$\mathrm{Pr} > 0.6$	$\mathrm{Sh} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Sc}^{1/3}$,	$\mathrm{Sc} > 0.5$
2. *Fully Developed Flow in Smooth Circular Pipes*			
(*a*) Laminar flow ($\mathrm{Re} < 2300$)			
$\mathrm{Nu} = 3.66$		$\mathrm{Sh} = 3.66$	
(*b*) Turbulent flow ($\mathrm{Re} > 10{,}000$)			
$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}$,	$0.7 < \mathrm{Pr} < 160$	$\mathrm{Sh} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Sc}^{0.4}$,	$0.7 < \mathrm{Sc}\ 160$

Diffusion of Water Vapor in Air:

Marrero and Mason empirical formula:

$$D_{H_2O-Air} = 1.87\times10^{-10}\,\frac{T^{2.072}}{P} \quad (m/s^2) \quad 280\,K < T < 450\,K$$

Table 2 D_{H_2O-Air} at 1 (atm)

T, °C	D_{H_2O-Air} or D_{Air-H_2O} at 1 atm, in m^2/s
0	2.09×10^{-5}
5	2.17×10^{-5}
10	2.25×10^{-5}
15	2.33×10^{-5}
20	2.42×10^{-5}
25	2.50×10^{-5}
30	2.59×10^{-5}
35	2.68×10^{-5}
40	2.77×10^{-5}
50	2.96×10^{-5}
100	3.99×10^{-5}
150	5.18×10^{-5}

Example 1:

Dry air at 15°C and 92 kPa flows over a 2-m-long wet surface with a free stream velocity of 4 m/s. Determine the average mass transfer coefficient.

Solution:

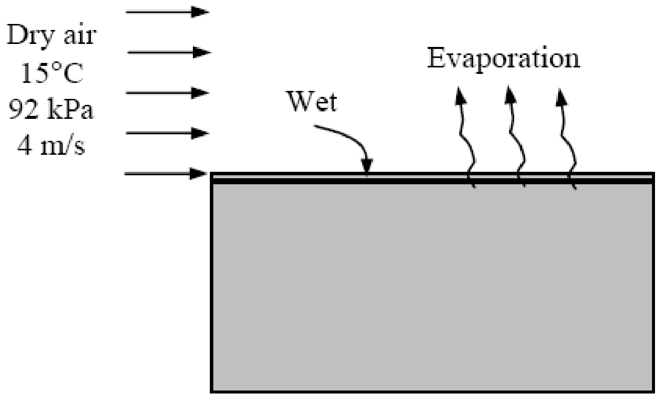
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 92 kPa = 92/101.325 = 0.908 atm, for which (Table A-15)

$$\nu = \nu_{1\,\text{atm}} / P(\text{atm}) = (1.47\times10^{-5}\ \text{m}^2/\text{s})/0.908\,\text{atm} = 1.62\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\,\frac{T^{2.072}}{P} = 1.87\times10^{-10}\,\frac{(288\ \text{K})^{2.072}}{0.908\ \text{atm}} = 2.57\times10^{-5}\ \text{m}^2/\text{s}$$



The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(4\ \text{m/s})(2\ \text{m})}{1.62\times10^{-5}\ \text{m}^2/\text{s}} = 493{,}827$$

which is less than 500,000, and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.62\times10^{-5}\ \text{m}^2/\text{s}}{2.57\times10^{-5}\ \text{m}^2/\text{s}} = 0.630$$

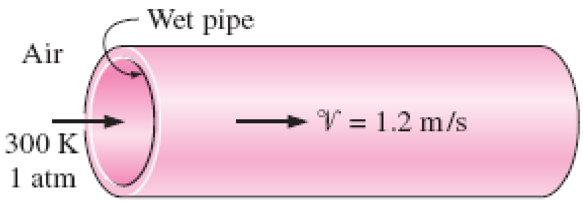
Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}^{0.5}\,\text{Sc}^{1/3} = 0.664(493{,}827)^{0.5}(0.630)^{1/3} = 400.1$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{L} = \frac{(400.1)(2.57\times10^{-5}\ \text{m}^2/\text{s})}{2\,\text{m}} = \mathbf{0.00514\,m/s}$$

Example 2:



Consider a circular pipe of inner diameter $D = 0.015$ m whose inner surface is covered with a layer of liquid water as a result of condensation (Fig. 14–49). In order to dry the pipe, air at 300 K and 1 atm is forced to flow through it with an average velocity of 1.2 m/s. Using the analogy between heat and mass transfer, determine the mass transfer coefficient inside the pipe for fully developed flow.

SOLUTION The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 300 K and 1 atm, for which $\nu = 1.58 \times 10^{-5}$ m²/s (Table A–15). The mass diffusivity of water vapor in the air at 300 K is determined from Eq. 14–15 to be

$$D_{AB} = D_{H_2O\text{-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{300^{2.072}}{1} = 2.54 \times 10^{-5}\ \text{m}^2/\text{s}$$

Analysis The Reynolds number for this internal flow is

$$\text{Re} = \frac{\mathcal{V} D}{\nu} = \frac{(1.2\ \text{m/s})(0.015\ \text{m})}{1.58 \times 10^{-5}\ \text{m}^2/\text{s}} = 1139$$

which is less than 2300 and thus the flow is laminar. Therefore, based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are Nu = Sh = 3.66. Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(3.66)(2.54 \times 10^{-5}\ \text{m}^2/\text{s})}{0.015\ \text{m}} = \mathbf{0.00620\ m/s}$$

The mass transfer rate (or the evaporation rate) in this case can be determined by defining the logarithmic mean concentration difference in an analogous manner to the logarithmic mean temperature difference.

Example 3:

A 2-in.-diameter spherical naphthalene ball is suspended in a room at 1 atm and 80°F. Determine the average mass transfer coefficient between the naphthalene and the air if air is forced to flow over naphthalene with a free stream velocity of 15 ft/s. The Schmidt number of naphthalene in air at room temperature is 2.35.

Solution:

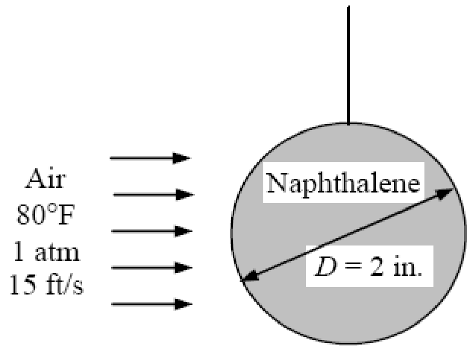
Properties The Schmidt number of naphthalene in air at room temperature is given to be 2.35. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm from Table A-15E,

$k = 0.015 \text{ Btu/h.ft.°F}$ $\qquad \nu = 0.17\times10^{-3} \text{ ft}^2/\text{s}$

$\mu = 1.250 \times 10^{-5} \text{ lbm/ft.s}$ $\qquad \text{Pr} = 0.72$

Analysis Noting that the Schmidt number for naphthalene in air is 2.35, the mass diffusivity of naphthalene in air is determined from

$$\text{Sc} = \frac{\nu}{D_{AB}} \longrightarrow D_{AB} = \frac{\nu}{\text{Sc}} = \frac{0.17\times10^{-3} \text{ ft}^2/\text{s}}{2.35} = 7.234\times10^{-5} \text{ ft}^2/\text{s}$$



The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(15 \text{ ft/s})(2/12 \text{ ft})}{(0.17\times10^{-3} \text{ ft}^2/\text{s})} = 14{,}706$$

Noting that $\mu_\infty = \mu_s$ for air in this case since the air and the ball are assumed to be at the same temperature, the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\text{Sh} = \frac{h_{mass} D}{D_{AB}} = 2 + \left[0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}\right]\text{Sc}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(14{,}706)^{1/2} + 0.06(14{,}706)^{2/3}\right](2.35)^{0.4} = 121$$

Then the mass transfer coefficient becomes

$$h_{mass} = \frac{\text{Sh} D_{AB}}{D} = \frac{(121)(7.234\times10^{-5} \text{ ft}^2/\text{s})}{0.166 \text{ ft}} = \mathbf{0.0525 \text{ ft/s}}$$

Discussion Note that the Nusselt number relations in heat transfer can be used to determine the Sherwood number in mass transfer by replacing Prandtl number by the Schmidt number.